Logical Characterizations of Phonological Patterns

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Introduction

- We introduce **logical characterizations** of language-specific phonological generalizations
- Ingredients of a logical characterization:
  - Representation (**model**)
  - Logic (**syntax**)
  - Relation between logic and representation (**semantics**)
- Excellent tool for both describing and explaining phonology

Why use logic?

- Constraints are **well-defined**; we know exactly the range of constraints and how to interpret them
- Logical constraints can apply to any well-defined structure
- The **computational nature** of logical characterizations are well-understood; they can be related to formal language/automata-theoretic/algebraic characterizations of patterns
- By understanding the kind of logical statements we need for phonology, we understand its computational nature, and how it might be learned

Local phonotactics (strings)

- $*N\r{C}$ (Pater, 2004)
- Quechua:
  - kamba   ‘yours’
  - *ka**mp**a

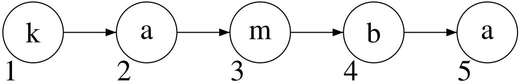
Local phonotactics (strings)

- **Model**: strings defined over immediate successor ($\triangleleft$) and some finite alphabet

$$\langle W, \triangleleft, P_a, P_b, ..., P_z \rangle$$

- kamba

$$\langle \quad \{1,2,3,4,5\}_W, \{(1,2),(2,3),(3,4),(4,5)\}_\triangleleft, \{2,5\}_a, \{1\}_k, \{3\}_m, \{4\}_p \quad \rangle$$



Local phonotactics (strings)

- **Syntax:** First order (FO) logic
  - Basic predicates: $x \triangleleft y$ and $a(x)$ for every a in alphabet
  - Boolean connectives: $\neg$, $\wedge$, $\vee$, $\rightarrow$
  - Quantifiers: $(\forall x_1, x_2, ..., x_n)$, $(\exists x_1, x_2, ..., x_n)$
- **Semantics:**
  - Variables range over positions in word
  - $(\forall x_1, x_2, ..., x_n)[\varphi(x_1, x_2, ..., x_n)]$
    "$\varphi(x_1, x_2, ..., x_n)$ must be true for all $x_1, x_2, ..., x_n$"
  - $(\exists x_1, x_2, ..., x_n)[\varphi(x_1, x_2, ..., x_n)]$
    "There must be some $x_1, x_2, ..., x_n$ for which $\varphi(x_1, x_2, ..., x_n)$ is true"

Local phonotactics (strings)

- *N$\mathring{\text{C}}$

$$nasal(x) \equiv m(x) \vee n(x) \vee \ldots \vee \eta(x)$$

$$voiceless(x) \equiv p(x) \vee t(x) \vee \ldots \vee k(x)$$

$$\varphi_{*\text{N}\mathring{\text{C}}} \equiv (\forall x, y)[(x \triangleleft y \wedge nasal(x)) \rightarrow \neg voiceless(y)]$$

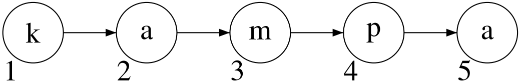
Local phonotactics (strings)

- *NC̥

$$nasal(x) \equiv m(x) \vee n(x) \vee ... \vee \eta(x)$$

$$voiceless(x) \equiv p(x) \vee t(x) \vee ... \vee k(x)$$

$$\varphi_{*\text{NC̥}} \equiv (\forall x, y)[(x \triangleleft y \wedge nasal(x)) \rightarrow \neg voiceless(y)]$$



Local phonotactics (strings)

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$$nasal(x) \equiv m(x) \vee n(x) \vee \ldots \vee \eta(x)$$

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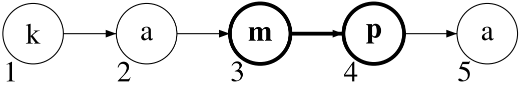


$\varphi_{*\text{NC̥}}$ is not true when $x = 3$ and $y = 4$

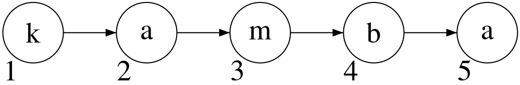
Local phonotactics (strings)

- *NC̥

$$nasal(x) \equiv m(x) \vee n(x) \vee \ldots \vee ŋ(x)$$

$$voiceless(x) \equiv p(x) \vee t(x) \vee \ldots \vee k(x)$$

$$\varphi_{*\mathrm{N}\underset{\circ}{\mathrm{C}}} \equiv (\forall x, y)[(x \triangleleft y \wedge nasal(x)) \rightarrow \neg voiceless(y)]$$



$\varphi_{*\mathrm{N}\underset{\circ}{\mathrm{C}}}$ is true for all values of x and y

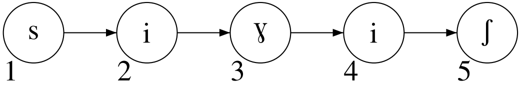
Long-distance phonotactics (strings)

- Sibilant harmony: *s...ʃ, *ʃ...s
- Navajo (Sapir and Hoijer, 1967):
  - sì-tí 'he is lying'
  - ʃì-ɣìʃ 'it is bent, curved'
  - ***s**ì-ɣìʃ

Long-distance phonotactics (strings)

- *s...ʃ
- Model:

$$\langle W, <, P_a, P_b, ..., P_z \rangle$$



Long-distance phonotactics (strings)

- *s…ʃ

$$+\mathit{AntSib}(x) \equiv \mathrm{s}(x) \vee \mathrm{z}(x) \vee \mathrm{ts}(x)$$

$$-\mathit{AntSib}(x) \equiv \mathrm{ʃ}(x) \vee \mathrm{ʒ}(x) \vee \mathrm{tʃ}(x)$$

$$\varphi_{*\mathrm{s...ʃ}} \equiv (\forall x, y)[(x < y \wedge +\mathit{AntSib}(x)) \rightarrow \neg - \mathit{AntSib}(y)]$$

Long-distance phonotactics (strings)

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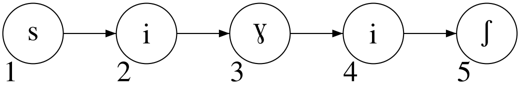

Long-distance phonotactics (strings)

- *s...ʃ

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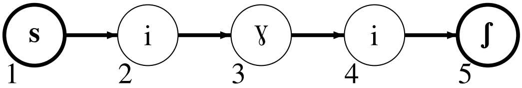


$\varphi_{*\text{s...ʃ}}$ is not true when $x = 1$ and $y = 5$

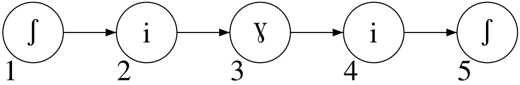
Long-distance phonotactics (strings)

- *s...ʃ

$$+AntSib(x) \equiv \mathrm{s}(x) \vee \mathrm{z}(x) \vee \mathrm{ts}(x)$$

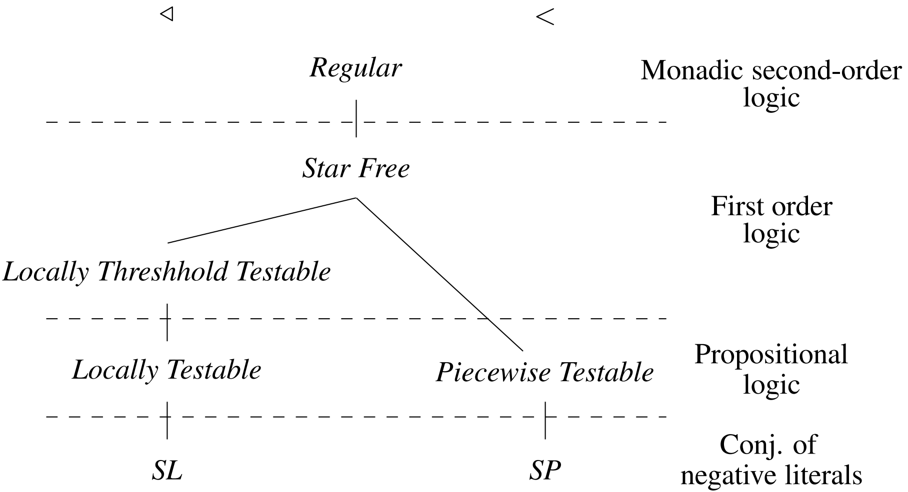
$$-AntSib(x) \equiv \mathrm{ʃ}(x) \vee \mathrm{ʒ}(x) \vee \mathrm{tʃ}(x)$$

$$\varphi_{*\mathrm{s...ʃ}} \equiv (\forall x, y)[(x < y \wedge +AntSib(x)) \rightarrow \neg - AntSib(y)]$$



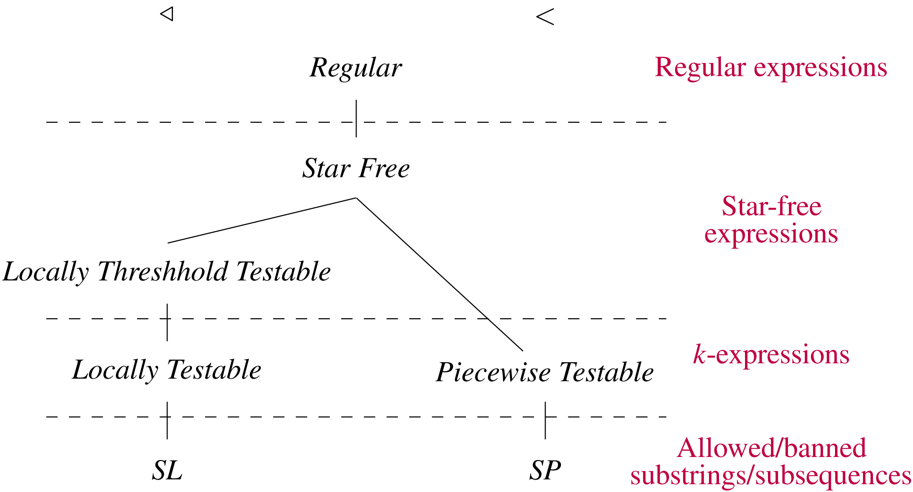
$\varphi_{*\mathrm{s...ʃ}}$ is true for all values of x and y

Subregular hierarchy



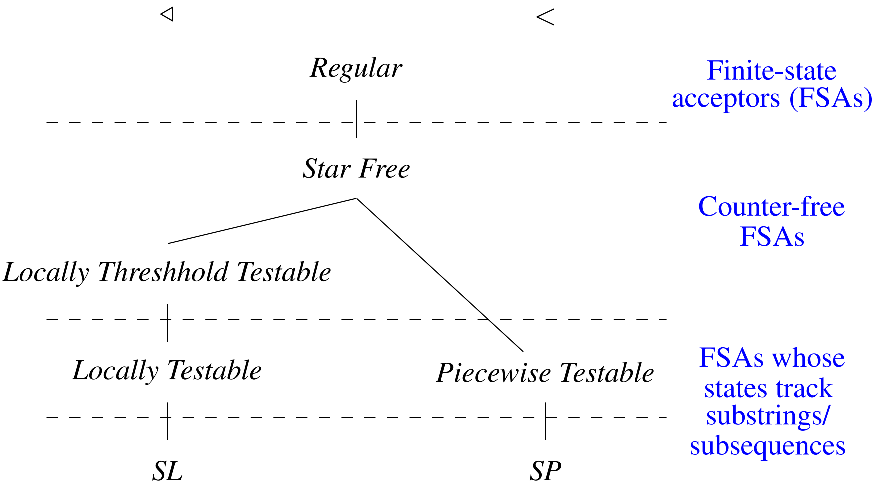
(Rogers et al., 2013; Rogers and Pullum, 2011)

Subregular hierarchy



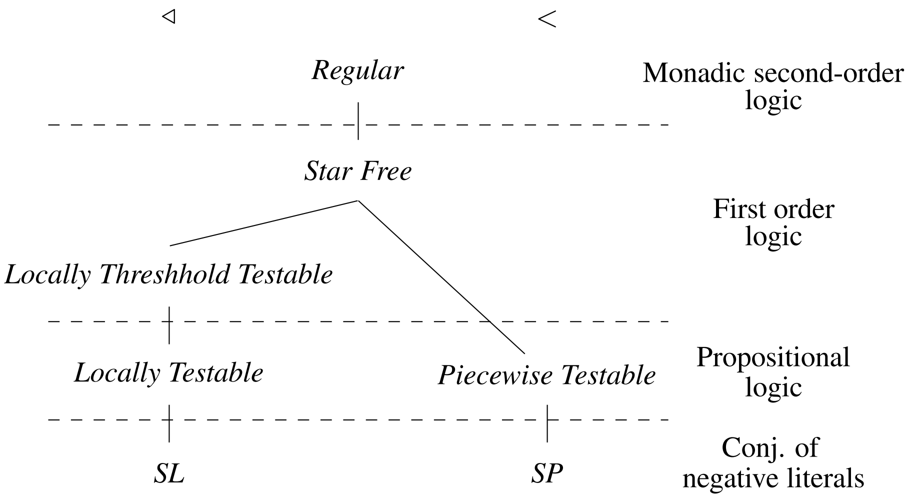
(Hopcroft et al., 2006; Rogers and Pullum, 2011)

Subregular hierarchy

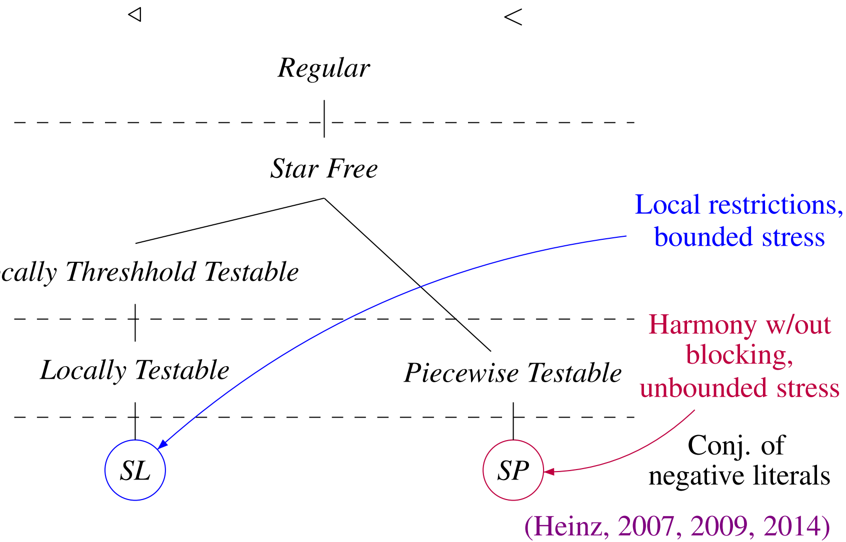


(McNaughton and Papert, 1971; Rogers et al., 2010)

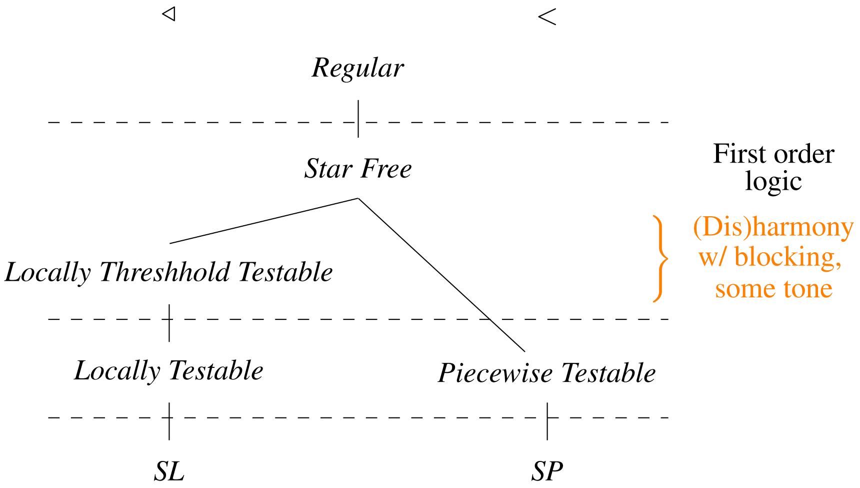
Subregular hierarchy and phonology



Subregular hierarchy and phonology



Subregular hierarchy and phonology



(Heinz et al. 2011; McMullin and Hansson, to appear; Jardine 2016)

Autosegmental representations

Mende word tone (Leben, 1973; Goldsmith, 1976)

a. kɔ́	H	'war'	b. pɛ́lɛ́	HH	'house'	c. háwámá	HHH	'waist'
d. kpà	L	'debt'	e. bɛ̀lɛ̀	LL	'pants'	f. kpàkàlì	LLL	'stool'
g. mbû	F	'owl'	h. ngílà	HL	'dog'	i. félàmà	HLL	'junction'
j. mbǎ	R	'rice'	k. nìká	LH	'cow'	l. ndàvúlá	LHH	'sling'
m. mba᷈	R-F	'comp.'	n. nyàhâ	LF	'woman'	o. nìkílì	LHL	'nut'

Autosegmental representations

Mende word tone (Leben, 1973; Goldsmith, 1976)

H	a. kɔ́	H	'war'	b. pɛ́lɛ́	HH	'house'	c. háwámá	HHH	'waist'
L	d. kpà	L	'debt'	e. bɛ̀lɛ̀	LL	'pants'	f. kpàkàlì	LLL	'stool'
HL	g. mbû	F	'owl'	h. ngílà	HL	'dog'	i. félàmà	HLL	'junction'
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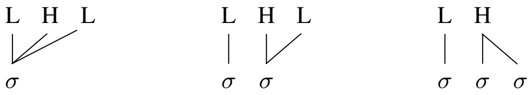
LHL	m. mbǎ̂	R-F	'comp.'	n. nyàhâ	LF	'woman'	o. nìkílì	LHL	'nut'

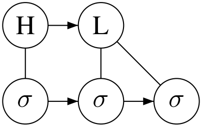
- Words choose between 5 melodies (*HLH)
- Plateaus and contours appear at the right edge of the word



Autosegmental representations

- Autosegmental representations are **graphs** (Goldsmith, 1976; Coleman and Local, 1991)

félàmà HLL 'junction'



- New predicate $x \circ y$ for (symmetric) association relation

$$\langle W, \triangleleft, \circ, P_a, P_b, ..., P_z \rangle$$

Autosegmental representations

Mende

- No HLH

- Multiple association at right edge

Autosegmental representations

Mende

- No HLH

$$(\forall x, y, z)[(x \triangleleft y \triangleleft z) \rightarrow \neg(H(x) \wedge L(y) \wedge H(z))]$$

- Multiple association at right edge

Autosegmental representations

Mende

- No HLH

$$(\forall x, y, z)[(x \triangleleft y \triangleleft z) \rightarrow \neg(H(x) \wedge L(y) \wedge H(z))]$$

- Multiple association at right edge

$$last(x) \equiv (\forall y)[\neg(x \triangleleft y)]$$

Autosegmental representations

Mende

- No HLH

$$(\forall x, y, z)[(x \triangleleft y \triangleleft z) \rightarrow \neg(H(x) \wedge L(y) \wedge H(z))]$$

- Multiple association at right edge

$$last(x) \equiv (\forall y)[\neg(x \triangleleft y)]$$

$$(\forall x, y, z)[(\neg last(x) \wedge x \circ y \wedge x \circ z) \rightarrow y = z]$$

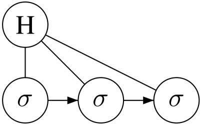
Autosegmental representations

Mende

- Multiple association at right edge

$$(\forall x, y, z)[(\neg \mathit{last}(x) \wedge x \circ y \wedge x \circ z) \rightarrow y = z]$$

háwámá
HHH
'waist'



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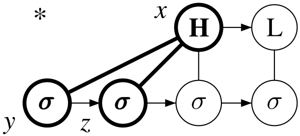
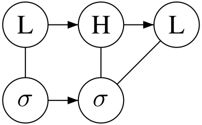
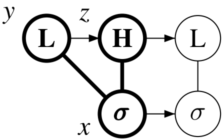


nyàhâ
LF
'woman'





Autosegmental representations

- Language-specific constraints are local in the logical sense; they use $\circ$ and $\triangleleft$, not $<$ (Jardine, 2016)
  - Hausa – multiple association only at left edge (Newman, 1986, 2000)

$$(\forall x, y, z)[(\neg \mathit{first}(x) \wedge x \circ y \wedge x \circ z) \rightarrow y = z]$$

  - Kukuya – H can only multiply associate if it is the only tone in the word (Zoll, 2003)

$$(\forall x, y, z)[(H(x) \wedge \neg \mathit{first}(x) \wedge \neg \mathit{last}(x) \wedge x \circ y \wedge x \circ z) \rightarrow y = z]$$

- In contrast, the 'universal' NCC requires $<$ (Coleman and Local, 1991)

$$(\forall x, y, u, v)[(x \circ u \wedge y \circ v \wedge x < y) \rightarrow \neg(v < u)]$$

Graph logic hierarchy

$\triangleleft$ $<$

Monadic second-order logic

(Courcelle et al., 2012)

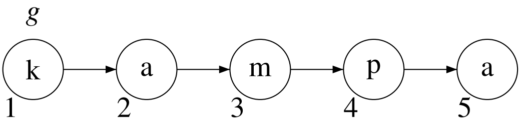
(Jardine, 2016)

Conj. of negative literals

Maps

- Post-nasal obstruent voicing
- Quechua (Pater, 2004): kampa $\mapsto$ kamba, ‘yours’

Underlying form



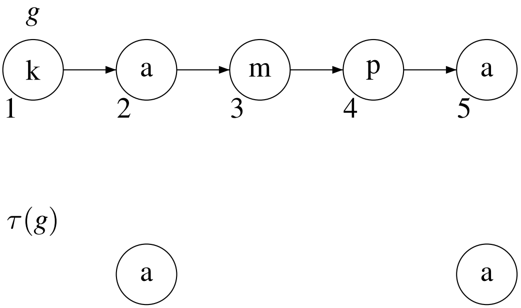
Surface form

- We define a map $\tau(g)$ that 'builds' an output graph for a given input graph.
- Logical formulae define the following in terms of the input graph:
  - What nodes exist in the output graph
  - The edges between nodes in the output graph
  - The labels of the nodes of the output graph

Surface form

- For each node in the input graph labeled with a vowel, there exists a corresponding node in the output graph labeled with that same vowel.
  - $\varphi_a^0(x) \equiv a(x)$

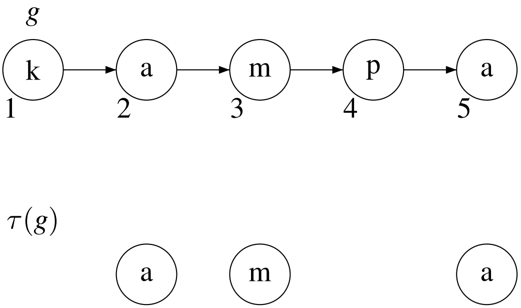
Surface form



Surface form

- Input nodes that are labeled with a nasal have output correspondents labeled with the same nasal.
  - $\varphi_m^0(x) \equiv m(x)$

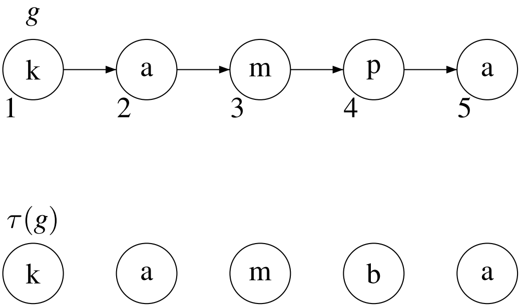
Surface form



Surface form

- For each voiceless obstruent in the input, a corresponding output node exists that is
  - voiced iff this predicate evaluates to true: $mp(x) \equiv p(x) \wedge \exists y[m(y) \wedge y \triangleleft x]$
  - voiceless iff $\neg mp(x)$
- $\varphi_b^0(x) \equiv b(x) \vee mp(x)$
- $\varphi_p^0(x) \equiv p(x) \wedge \neg mp(x)$

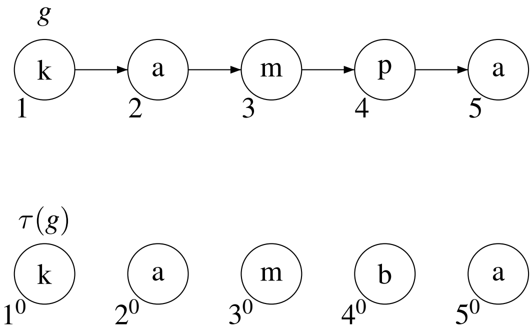
Surface form



Node formulae

- $\varphi_a^0(x) \equiv a(x)$
- $\varphi_m^0(x) \equiv m(x)$
- $\varphi_b^0(x) \equiv b(x) \vee mp(x)$
- $\varphi_p^0(x) \equiv p(x) \wedge \neg mp(x)$

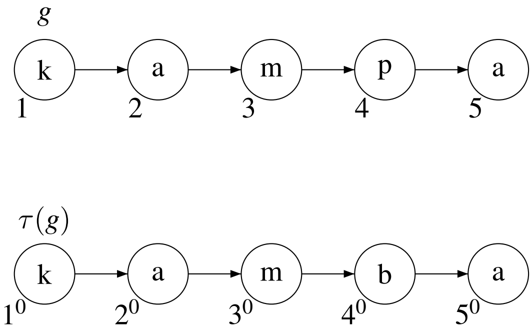
Copy set



Edge formulae

- In this example, edges are preserved:
  - $\varphi_*^{0,0}(x, y) \equiv \texttt{true}$

Preserving edges



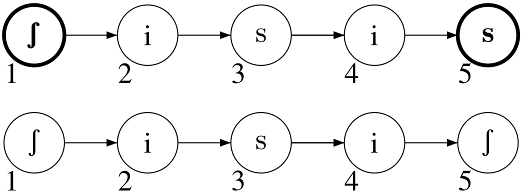
How powerful is the logic we need for phonology?

- FO is powerful enough to capture some major generalizations
- But it, and other powerful logics, are *too* powerful for phonology

How powerful is the logic we need for phonology?

- ‘First-last’ harmony — unattested and difficult to learn (Lai, 2012, 2015)

$$\varphi_{\text{FL}} \equiv (\forall x, y)[(\textit{first}(x) \wedge \textit{last}(y) \wedge -\textit{AntSib}(x)) \rightarrow -\textit{AntSib}(y)]$$



How powerful is the logic we need for phonology?

- **Requiring** structures
  "Have three [-ant] sibilants"

  $$(\exists x, y, z)[-AntSib(x) \wedge -AntSib(y) \wedge -AntSib(z) \wedge x \neq y \neq z]$$

- Courcelle et al. (2012): MSO can define abstract graph properties; ex. planarity, connectedness, 3-colorability

How powerful is the logic we need for phonology?

- What is a restrictive characterization of phonological FO statements?
- What is the ‘graph subregular’ hierarchy for non-linear representations?
- What is the corresponding hierarchy for transformations?

Open questions and low-hanging fruit

- How do we incorporate features? How expressive/restrictive are feature-based models?

$$\langle W, \triangleleft, P_{+F}, P_{-F}, ..., P_{velar} \rangle$$

- What about correspondence models (Rose and Walker, 2004; Hansson, 2001; Shih and Inkelas, 2014)? Metrical phonology (Hayes, 1995)?
- What are explicit definitions of often-used constraints (Potts and Pullum, 2002; Eisner, 1997; Riggle, 2004; Graf, 2010; de Lacy, 2011)? What is their computational nature (Graf, 2010)?

Learnability

- Phonological learning research in a variety of frameworks has emphasized the need for restrictiveness.
  - Constraint-based grammars (Tesar, 2014; Magri and Kager, 2015)
  - Finite-state phonology (Gildea and Jurafsky, 1996; Chandlee et al., 2014; Jardine et al., 2014)
- What role do the restrictions on FO logic play in learning?

Acknowledgments

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